Antiderivatives

Part 1

What do you think antiderivative means?

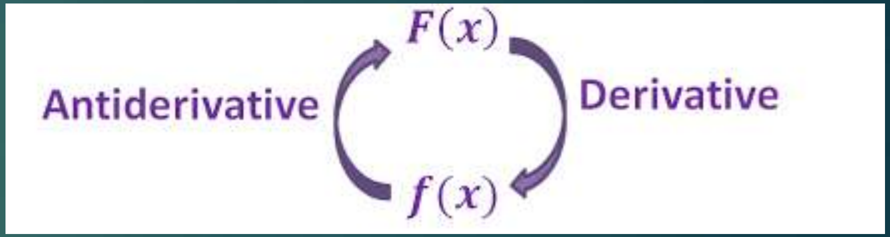
- Suppose we have two functions f and g

the derivative of f is g

$f'(x) = g(x)$

f is a function whose derivative is g

f is an antiderivative of g



Definition

Read the definition very well

- Let f be a function and continuous over an interval I.
- g is called an antiderivative (primitive) of f if g is defined on I satisfying $g'(x) = f(x)$.
- Denoted by $\int f(x)dx$: the set of all antiderivatives of f

Exercise

let f & g be 2 functions defined on $I =]-3, +\infty[$ by $f(x) = \frac{x-2}{x+3}$ & $g(x) = \frac{5}{(x+3)^2}$

a) prove that f is an antiderivative of g.

Pause the video and solve this exercise

Solution

- $f'(x) = \left(\frac{x-2}{x+3}\right)' = \frac{x+3-x+2}{(x+3)^2} = \frac{5}{(x+3)^2} = g(x)$

- **Then f is an antiderivative of g**

Can you deduce another antiderivative of g ?

$\frac{x-2}{x+3}+1$

$\frac{x-2}{x+3}-5$

$\frac{x-2}{x+3}-\sqrt{7}$

$\frac{x-2}{x+3}+c$

So,

$$\int g(x)dx = \frac{x-2}{x+3}+c$$

(c is any real number)

Primitives of usual functions

- $\int dx = \ ...$
- $\int 2dx = \ ...$
- $\int -4dx = \cdots$

- $\int dx = x + c$
- $\int 2dx = 2x + c$
- $\int -4dx = -4x + c$

First Rule

$$\int k\,dx = kx + c$$

k and c are two real numbers

Primitives of usual functions

- $\int xdx = \; ...??$ What is the function whose derivative is x ??

$$\int xdx = \int \frac{2xdx}{2} = \frac{x^2}{2} + c$$

- $\int x^2dx = \; ...??$ What is the function whose derivative is x^2 ??

$$\int x^2dx = \int \frac{3x^2dx}{3} = \frac{x^3}{3} + c$$

General Rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

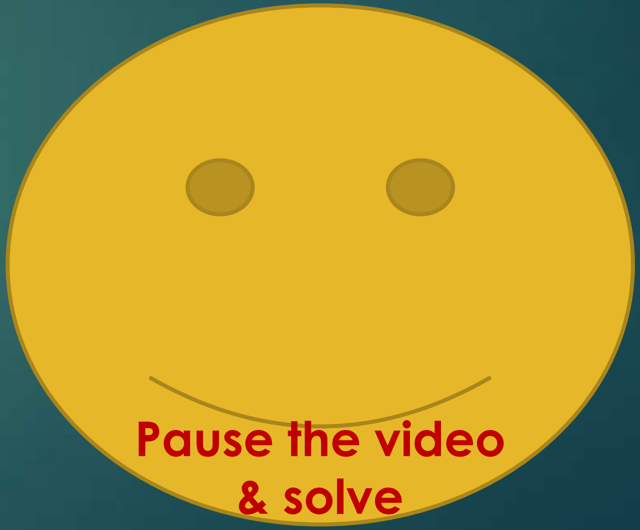
$(n \neq -1)$

Application : Find the following antiderivatives

- $\int \frac{1}{x^2} dx$
- $\int \sqrt{x} dx$
- $\int 10t^4 dt$
- $\int \frac{6}{x^5} dx$

Hint

- $\frac{1}{x^2} = x^{-2}$
- $\sqrt{x} = x^{\frac{1}{2}}$



Check your answers now

- $\int \frac{1}{x^2} dx = \int x^{-2} dx = \frac{x^{-2+1}}{-2+1} + c = \frac{x^{-1}}{-1} + c = -\frac{1}{x} + c$
- $\int \sqrt{x} dx = \int x^{\frac{1}{2}} dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c = \frac{2}{3}\sqrt{x^3} + c$
- $\int 10t^4 dt = 10\frac{t^5}{5} + c = 2t^5 + c$
- $\int \frac{6}{x^5} dx = \int 6x^{-5} dx = \frac{6x^{-4}}{-4} + c = -\frac{3}{2x^4} + c$

Property

$$\int [f(x) \mp h(x)]dx = \int f(x)dx \mp \int h(x)dx$$

Important Notes

$$\int f(x) \times g(x)dx \neq \int f(x)dx \times \int g(x)\,dx$$

$$\int (2x+1)x^3dx \neq \int (2x+1)dx \times \int x^3dx$$

You should expand then integrate

$$\int (2x^4+x^3)dx$$

$$\int \frac{f(x)}{g(x)} \neq \frac{\int f(x)dx}{\int g(x)dx}$$

$$\int \frac{x^4+x^3-1}{x^2}dx \neq \frac{\int (x^4+x^3-1)dx}{\int x^2dx}$$

$$\int \frac{x^4+x^3-1}{x^2}dx = \int (x^2+x^1-x^{-2})dx$$

Exercise:

- $\int x^2(x^3+5)dx$
- $\int (x+2)^2 dx$
- $\int \frac{x^4+3}{x^2} dx$
- $\int \frac{\sqrt{x}+3}{x^2} dx$

Just For Fun